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Tutorial Sheet

Fourier transforms:

Fourier transforms, Fourier sine and Fourier cosine transforms and their properties.

Fourier transforms:

1. Find the Fourier transform of the following function

(i) $F(x)=e^{-|x|}$ (ii) $F(x)=\frac{1}{2\varepsilon},|x|\leq\varepsilon$

Ans: (i) $\frac{2}{1+p^2}$ (ii) $\frac{\sin p\varepsilon}{p\varepsilon}$

2. Find the Fourier transform of the given function $F(x)=\frac{\sin ax}{x}, a>0$.

Ans: Case 1 : f(p)= π and Case 2: f(p) =0

3. Find the complex Fourier transform of dirac delta function $\delta(x-a)$.

Ans: e^{ipa}

4. Find the inverse Fourier transform of f(p)=$e^{-|p|y}$.

Ans: $\frac{y}{\pi(y^2+x^2)}$

5. Find Fourier sine transformation of $\frac{e^{-ax}}{x}, a>0$ hence find Fourier sine transformation of $\frac{1}{x}$.

Ans: $\frac{\pi}{2}$

6. Find the Fourier cosine transformation of $\frac{1}{1+x^2}$ and hence find Fourier sign transformation of $\frac{x}{1+x^2}$.

Ans: $\frac{\pi}{2}e^{-p}$

7. Find the Fourier cosine transformation of e^{-x^2}.

Ans: $\frac{\sqrt{\pi}}{2}e^{-\frac{p^2}{4}}$.

8. Solve the integral equations: $\int_0^\infty f(x)\cos\lambda x dx = e^{-\lambda}$.

Ans: F(x)= $\frac{2}{\pi x}(1+\cos x-2\cos 2x)$

9. Find F(X) if its Fourier sine transformation is $\frac{\pi}{2}$.

Ans: $\frac{1}{x}$

10. Find Fourier sine and cosine transformation of $\frac{1}{\sqrt{x}}$.

Ans: $\sqrt{\frac{\pi}{2p}},\sqrt{\frac{\pi}{2p}}$

11. Find Fourier finite sine transformation of $F(x)=1-\frac{x}{\pi}$.

Ans: $\frac{1}{p^2-k^2}, k\neq 0,1,2,.....$

12. Prove that : $\int_0^\infty\left(\frac{\sin x}{x}\right)^4 dx=\frac{\pi}{3}$

13. Using Parseval's identity, show that $\int_0^\infty \frac{x^2}{\left(a^2+x^2\right)\left(b^2+x^2\right)}dx=\frac{\pi}{2(a+b)}$.

14. Find Fourier finite cosine transformation of $\sin nx, n\in I$

Ans: $\frac{2n}{n^2-p^2};\left(n-p\right)$ is odd and 0 if even

15. State and prove the Convolution Theorem.